des lanières de tore

```
> restart;
> with(plots):
Warning, the name changecoords has been redefined
> R:=5;r:=2;
```

$$R := 5 \qquad (1.1)$$

$$r := 2$$

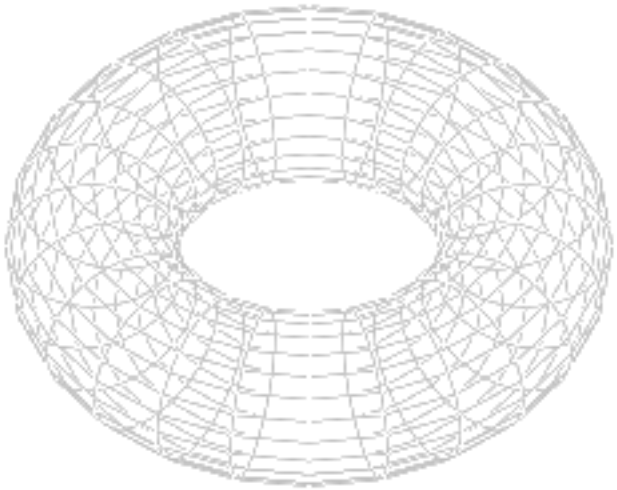
```
> P:=(theta,alpha)->[(R+r*cos(alpha))*cos(theta),(R+r*cos
  (alpha))*sin(theta),r*sin(alpha)];
```

$$P := (\theta, \alpha) \rightarrow [(R + r\cos(\alpha))\cos(\theta), (R + r\cos(\alpha))\sin(\theta), r\sin(\alpha)] \qquad (1.2)$$

```
> Q:=(theta)->[(R)*cos(theta),(R)*sin(theta),0];
```

$$Q := \theta \rightarrow [R\cos(\theta), R\sin(\theta), 0] \qquad (1.3)$$

```
> pneu:=plot3d(P(theta,t),theta=0..2*Pi,t=0..2*Pi,style=
  wireframe,color=gray):
> display(pneu,scaling=constrained,view=[-R-r..R+r,-R-r..R+r,-
  R-r..R+r]);
```



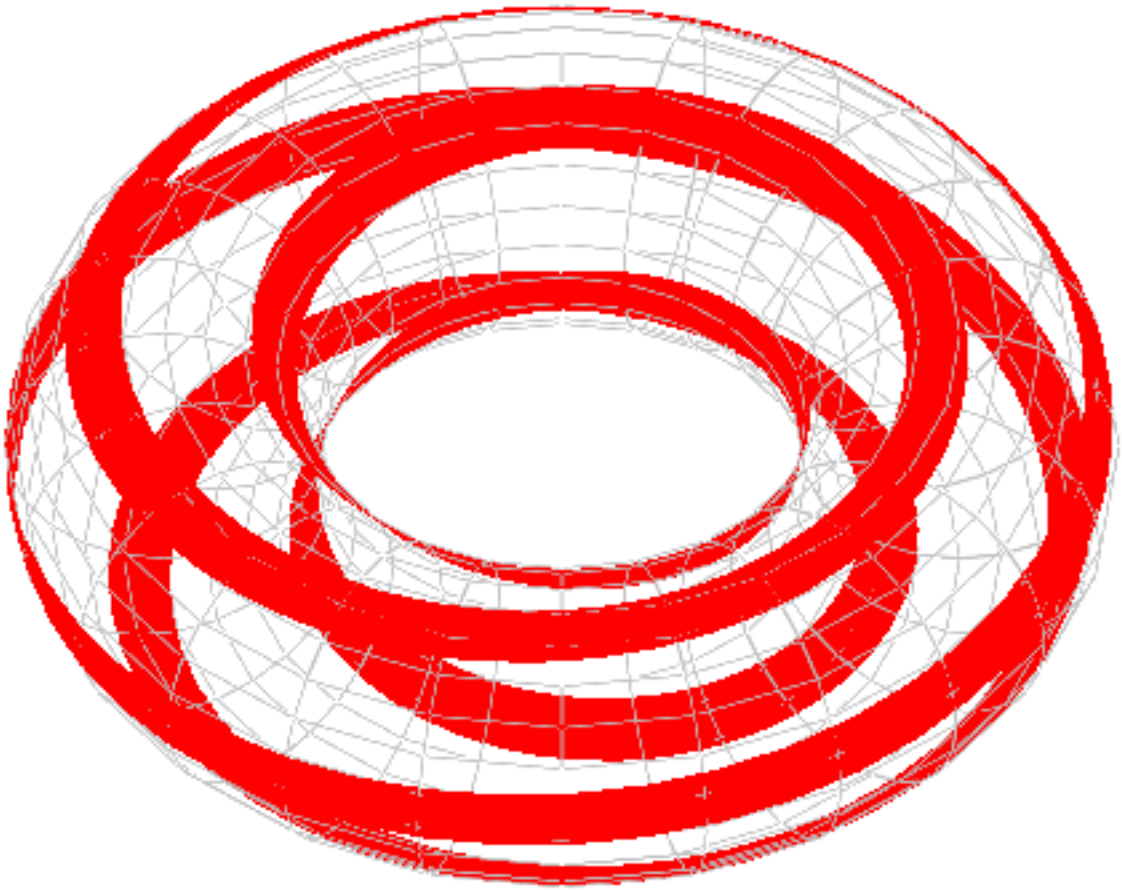
```
> N0:=u->Q(u);
```

$$N0 := Q \tag{1.4}$$

```
> N52:=(u,v)->P(5*u,2*u+v);
```

$$N52 := (u, v) \rightarrow P(5\,u, 2\,u + v) \tag{1.5}$$

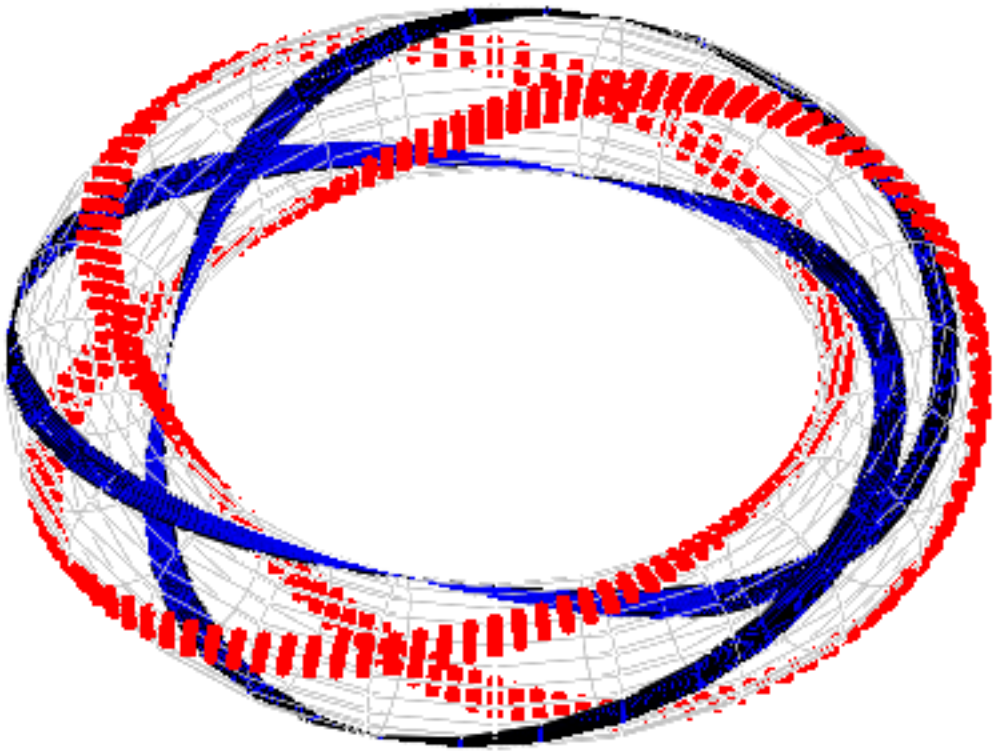
```
> bande0:=spacecurve(N0(u),u=0..2*Pi,color=blue,numpoints=5000)
  :
> bande1:=plot3d(N52(u,v),u=0..2*Pi,v=-0.2..0.2,color=red,
  numpoints=54321,style=patchnogrid):
> display(pneu,bande1,scaling=constrained,view=[-R-r..R+r,-R-r.
  .R+r,-R-r..R+r]);
```



```
> N2 := proc (u, v) options operator, arrow; P(Pi/3+2*u,3*u+v)
  end proc;
```

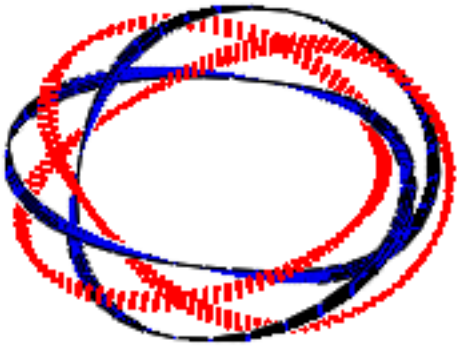
$$N2 := (u, v) \rightarrow P\left(\frac{1}{3}\,\pi + 2\,u, 3\,u + v\right) \tag{1.6}$$

```
> bande2:=plot3d(N2(u,v),u=0..2*Pi,v=-0.2..0.2,color=blue,
  numpoints=5600):
> display(pneu,bande1,bande2,scaling=constrained,view=[-R-r..R+
  r,-R-r..R+r,-R-r..R+r]);
```



```
> display(bande1,bande2,scaling=constrained,view=[-R-r..R+r,-R-
```

```
r..R+r,-R-r..R+r]);
```



```
> display(bande2,bande0,scaling=constrained,view=[-R-r..R+r,-R-
r..R+r,-R-r..R+r]);
```

```
> C1:=u->P(2*u,3*u);C2:=u->P(Pi/3+2*u,3*u);
```

$$C1 := u \to P(2\,u, 3\,u) \qquad (1.7)$$

$$C2 := u \to P\left(\frac{1}{3}\,\pi + 2\,u, 3\,u\right)$$

```
> V:=(u,v)->C1(u)-C2(v);
```

$$V := (u, v) \to C1(u) - C2(v) \qquad (1.8)$$

```
> V(u,v);
```

$$\left[-(5+\cos(3\,v))\cos\left(\frac{1}{3}\,\pi + 2\,v\right) + (5+\cos(3\,u))\cos(2\,u),\ -(5+\cos(3\,v))\sin\left(\frac{1}{3}\,\pi + 2\,v\right) + (5+\cos(3\,u))\sin(2\,u),\ -\sin(3\,v) + \sin(3\,u)\right] \qquad (1.9)$$

```
> V(1,2);
```

$$\left[-(5+\cos(6))\cos\left(\frac{1}{3}\,\pi + 4\right) + (5+\cos(3))\cos(2),\ -(5+\cos(6))\sin\left(\frac{1}{3}\,\pi + 4\right) + (5+\cos(3))\sin(2),\ -\sin(6) + \sin(3)\right] \qquad (1.10)$$

```
> norme:=proc(v) local a,b,c;a:=v[1];b:=v[2];c:=v[3]; sqrt(a^2+
  b^2+c^2) end;
```

$$norme := \mathbf{proc}(v) \qquad (1.11)$$

$$\quad \mathbf{local}\ a, b, c;\ a := v[1];\ b := v[2];\ c := v[3];\ \mathrm{sqrt}(a\text{^}2 + b\text{^}2 + c\text{^}2)$$

$$\mathbf{end\ proc}$$

```
> V_x:=diff(V(x,y),x);
```

$$V\_x := [\,-3\sin(3x)\cos(2x) - 2\,(5+\cos(3x))\sin(2x),\ -3\sin(3x)\sin(2x) + 2\,(5+\cos(3x))\cos(2x),\ 3\cos(3x)\,] \tag{1.12}$$

```
> V_y:=diff(V(x,y),y);
```

$$V\_y := \left[3\sin(3y)\cos\left(\frac{1}{3}\pi + 2y\right) + 2\,(5+\cos(3y))\sin\left(\frac{1}{3}\pi + 2y\right),\right. \tag{1.13}$$
$$\left. 3\sin(3y)\sin\left(\frac{1}{3}\pi + 2y\right) - 2\,(5+\cos(3y))\cos\left(\frac{1}{3}\pi + 2y\right),\ -3\cos(3y)\right]$$

```
> pv:=proc(u,v) [u[2]*v[3]-u[3]*v[2],u[3]*v[1]-u[1]*v[3],u[1]*v
  [2]-u[2]*v[1]] end;
```

$$pv := \mathbf{proc}(u, v) \tag{1.14}$$
$$[u[2]*v[3] - u[3]*v[2],\ u[3]*v[1] - u[1]*v[3],\ u[1]*v[2] - u[2]*v[1]]$$
$$\mathbf{end\ proc}$$

```
> ps:=proc(u,v) u[1]*v[1]+u[2]*v[2]+u[3]*v[3] end;
```

$$ps := \mathbf{proc}(u, v)\ u[1]*v[1] + u[2]*v[2] + u[3]*v[3]\ \mathbf{end\ proc} \tag{1.15}$$

```
> num:=ps(V(x,y),pv(V_x,V_y));
```

$$num := \left(-(5+\cos(3y))\cos\left(\frac{1}{3}\pi + 2y\right) + (5+\cos(3x))\cos(2x)\right)\left(-3\,(\right. \tag{1.16}$$
$$-3\sin(3x)\sin(2x) + 2\,(5+\cos(3x))\cos(2x))\cos(3y)$$
$$-3\cos(3x)\left(3\sin(3y)\sin\left(\frac{1}{3}\pi + 2y\right) - 2\,(5+\cos(3y))\cos\left(\frac{1}{3}\pi + 2y\right)\right)\Big)$$
$$+\left(-(5+\cos(3y))\sin\left(\frac{1}{3}\pi + 2y\right) + (5\right.$$
$$\left.+\cos(3x))\sin(2x)\right)\left(3\cos(3x)\left(3\sin(3y)\cos\left(\frac{1}{3}\pi + 2y\right) + 2\,(5\right.\right.$$
$$\left.\left.+\cos(3y))\sin\left(\frac{1}{3}\pi + 2y\right)\right) + 3\,(-3\sin(3x)\cos(2x) - 2\,(5\right.$$
$$\left.+\cos(3x))\sin(2x))\cos(3y)\right) + (-\sin(3y) + \sin(3x))\left((-3\sin(3x)\cos(2x)\right.$$
$$-2\,(5+\cos(3x))\sin(2x))\left(3\sin(3y)\sin\left(\frac{1}{3}\pi + 2y\right) - 2\,(5\right.$$
$$\left.+\cos(3y))\cos\left(\frac{1}{3}\pi + 2y\right)\right) - (-3\sin(3x)\sin(2x) + 2\,(5\right.$$
$$\left.+\cos(3x))\cos(2x))\left(3\sin(3y)\cos\left(\frac{1}{3}\pi + 2y\right) + 2\,(5+\cos(3y))\sin\left(\frac{1}{3}\pi\right.\right.$$
$$\left.\left.+2y\right)\right)\Big)$$

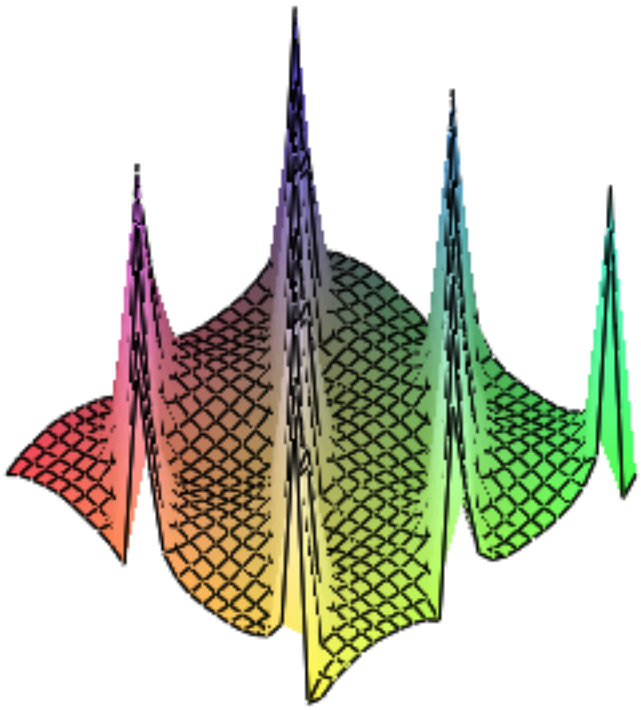
```
> den:=ps(V(x,y),V(x,y))^(3/2);
```

$$den := \left(\left(-(5+\cos(3y))\cos\left(\frac{1}{3}\pi+2y\right)+(5+\cos(3x))\cos(2x)\right)^2+\left(-(5+\cos(3y))\sin\left(\frac{1}{3}\pi+2y\right)+(5+\cos(3x))\sin(2x)\right)^2+(-\sin(3y)+\sin(3x))^2\right)^{3/2} \quad (1.17)$$

```
> fonc:=unapply(evalf(num/den),(x,y));
```

$$fonc := (x,y) \rightarrow ((-1.(5.+\cos(3.y))\cos(1.047197551+2.y)+(5.+\cos(3.x))\cos(2.x))(-3.(-3.\sin(3.x)\sin(2.x)+2.(5.+\cos(3.x))\cos(2.x))\cos(3.y)-3.\cos(3.x)(3.\sin(3.y)\sin(1.047197551+2.y)-2.(5.+\cos(3.y))\cos(1.047197551+2.y)))+(-1.(5.+\cos(3.y))\sin(1.047197551+2.y)+(5.+\cos(3.x))\sin(2.x))(3.\cos(3.x)(3.\sin(3.y)\cos(1.047197551+2.y)+2.(5.+\cos(3.y))\sin(1.047197551+2.y))+3.(-3.\sin(3.x)\cos(2.x)-2.(5.+\cos(3.x))\sin(2.x))\cos(3.y))+(-1.\sin(3.y)+\sin(3.x))((-3.\sin(3.x)\cos(2.x)-2.(5.+\cos(3.x))\sin(2.x))(3.\sin(3.y)\sin(1.047197551+2.y)-2.(5.+\cos(3.y))\cos(1.047197551+2.y))-1.(-3.\sin(3.x)\sin(2.x)+2.(5.+\cos(3.x))\cos(2.x))(3.\sin(3.y)\cos(1.047197551+2.y)+2.(5.+\cos(3.y))\sin(1.047197551+2.y))))\Big/((-1.(5.+\cos(3.y))\cos(1.047197551+2.y)+(5.+\cos(3.x))\cos(2.x))^2+(-1.(5.+\cos(3.y))\sin(1.047197551+2.y)+(5.+\cos(3.x))\sin(2.x))^2+(-1.\sin(3.y)+\sin(3.x))^2)^{3/2} \quad (1.18)$$

```
> plot3d(fonc(x,y),x=0..2*Pi,y=0..2*Pi);
```



```
> Digits:=4;
```

$$Digits := 4 \qquad (1.19)$$

```
> evalf(fonc(1,2));
```

$$0.1148 \qquad (1.20)$$

```
> 6*evalf(int(evalf(int(evalf(fonc(x,y)),x=0..Pi)),y=0..2*Pi/3)
  );
```

$$-394. \qquad (1.21)$$

```
> evalf(int(evalf(fonc(x,1)),x=0..Pi));
```

$$8.066 \qquad (1.22)$$

```
> val:=map(m->evalf(int(evalf(fonc(x,m)),x=0..Pi)),map(k->
  (k+0.5)*2*Pi/30,[$0..9]));
```

$$val := [4.194, 4.366, 5.181, 6.521, 7.831, 8.266, 7.592, 6.390, 5.243, 4.412] \qquad (1.23)$$

```
> convert(val,`+`)*2*Pi/30;;
```

$$3.999\,\pi \qquad (1.24)$$

```
> 6*%;
```

$$23.99\,\pi \qquad (1.25)$$

```
> evalf(%/(4.*Pi));
```

$$5.998 \qquad (1.26)$$

```
> N23:=(u,v)->P(2*u,3*u+v);
```

(1)

$$N23 := (u, v) \rightarrow P(2\,u, 3\,u + v) \quad \textbf{(1)}$$

```
> M23:=(u,v)->P(2*u,3*u+v+1.57);
```

$$M23 := (u, v) \rightarrow P(2\,u, 3\,u + v + 1.57) \quad \textbf{(2)}$$

```
> bande1:=plot3d(N23(u,v),u=0..2*Pi,v=-0.2..0.2,color=red,
  numpoints=54321,style=patchnogrid):
> bande2:=plot3d(M23(u,v),u=0..2*Pi,v=-0.2..0.2,color=green,
  numpoints=54321,style=patchnogrid):
> display(pneu,bande1,bande2,scaling=constrained,view=[-R-r..R+r,-
  R-r..R+r,-R-r..R+r]);
```